3rd International Hybrid Power Systems Workshop | Tenerife, Spain | 08 – 09 May 2018

An Adaptive Model with Joint Chance Constraints for a Hybrid Wind-Conventional Generator System

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Contents

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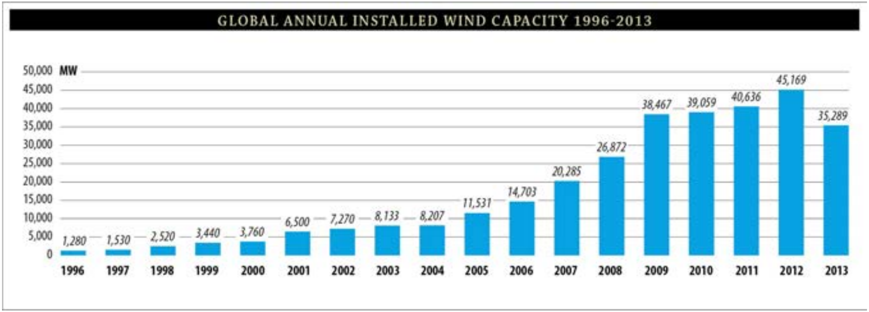
Wind energy



Source: Global Wind Energy Council

Wind energy

Challenges

- High initial investment costs
- Noise pollution from wind-turbines
- Intermittent and unreliable, or “non-dispatchable”

Dispatchable energy

- A reliable supplier of energy
- Provides load matching, cover for intermittent sources
- Examples of dispatchable plants: hydroelectricity, biomass, coal plants, concentrated solar (semi-dispatchable), nuclear, natural gas

Dispatchable energy

- A reliable supplier of energy
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- Examples of dispatchable plants: hydroelectricity, biomass, coal plants, concentrated solar (semi-dispatchable), nuclear, natural gas
- Wind is highly intermittent and not dispatchable

Introduction to the problem

Description:

- Bid a promised amount of energy for the day-ahead market
- Provide that energy using (co-located) wind and conventional generator

Introduction to the problem

Description:

- Bid a promised amount of energy for the day-ahead market
- Provide that energy using (co-located) wind and conventional generator
- Large penalty for not meeting promise
- Wind is cheap but highly stochastic

Not looking at: ancillary services, spinning reserve, intra-day market

Contents

Two optimization models:

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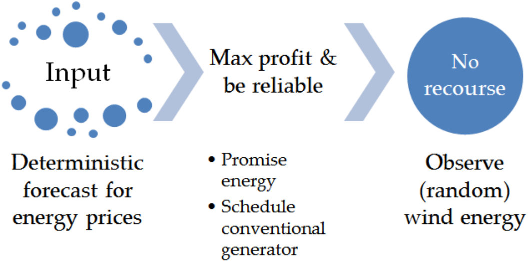
Decisions for each hour: (i) how much energy to promise, and (ii) how much energy to schedule from conventional generator

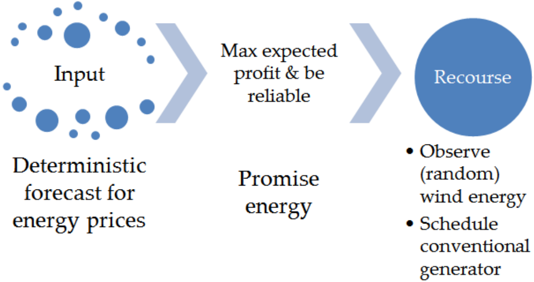


Non-adaptive model

- Day-ahead dispatch decisions
- Example: coal plant

Two optimization models

Decisions for each hour: (i) how much energy to promise, and (ii) how much energy to schedule from conventional generator



Input

Deterministic forecast for energy prices

Max expected profit & be reliable

Promise energy

Recourse

- Observe (random) wind energy
- Schedule conventional generator

Adaptive model

- Real-time dispatch decisions
- Example: natural gas plant

Joint chance constraints (JCC)

JCC

$$\mathbb{P}(f(x, y(\xi)) \leq 0) \geq 1 - \varepsilon$$

- First stage decision x, then an uncertainty, then a second stage decision $y(\xi)$
- Possibly dependency between uncertainty
- Computationally challenging
- Theoretically NP-hard

Optimization model

Sets

T	Set of time periods (hours) $\{1, 2, \ldots, \lvert T\rvert\}$
Ω	Set of wind energy scenarios $\{\omega_1, \omega_2, \ldots, \omega_{\lvert\Omega\rvert}\}$

Parameters

B_t	Operation cost of generator at time t (\$/MWh)
R_t	Market clearing price at time t (\$/MWh)
w_t^ω	Wind energy available from the farm under scenario ω at time t (MWh)
p^ω	Probability of scenario ω ($p^\omega = 1/N$ under SAA [1])
ε	Threshold on probability of failing to meet promised energy output
Δ	Hourly ramp of conventional generator (MWh)
M_t^ω	Sufficiently large positive number for an integer programming big M formulation
U	Minimum number of time periods required for generator to be on before it can be turned off (hours)
V	Minimum number of time periods required for generator to be off before it can be turned on (hours)
G	Maximum hourly output of generator if on (MWh)
g	Minimum hourly output of generator if on (MWh)

---

[1] Sample Average Approximation

Optimization model (contd.)

Decision Variables

x_t	Promised energy output to be delivered at time t (MWh)
y_t^ω	Energy above the minimum hourly output from the generator at time t under scenario ω (MWh)
z^ω	Takes value 1 if the promise is not met under scenario ω and takes value 0 otherwise
q_t^ω	Takes value 1 if the promise is not met for all scenarios with wind-energy values at least as large as scenario ω's value at time t, and takes value 0 otherwise
r_t^ω	On/off status of generator at time t (1 if on, else 0)
u_t^ω	Start-up status of generator at time t (1 if switched on, else 0)
v_t^ω	Shutdown status of generator at time t (1 if switched off, else 0)

Optimization model (contd.)

Adaptive model:

$$\max_{x,y,r,u,v} \quad \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t(y_t^\omega + gr_t^\omega)]) \tag{2a}$$

$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + gr_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \tag{2b}$$

$$x_t \geq 0, \forall t \in T \tag{2c}$$

$$(y^\omega, r^\omega, u^\omega, v^\omega) \in Y, \forall \omega \in \Omega \leftarrow \text{generator operating constraints} \tag{2d}$$

Optimization model (contd.)

Adaptive model:

$$\begin{aligned}
\max_{x,y,r,u,v} \quad & \sum_{t\in T} (R_t x_t - \mathbb{E}[B_t(y_t^\omega + gr_t^\omega)]) && \text{(2a)}\\
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\end{aligned}$$

Non-adaptive model:

$$\begin{aligned}
\max_{x,y,r,u,v} \quad & \sum_{t\in T} (R_t x_t - B_t(y_t + gr_t)) && \text{(3a)}\\
\text{s.t.} \quad & \mathbb{P}(y_t + gr_t + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon && \text{(3b)}\\
& x_t \geq 0, \forall t \in T && \text{(3c)}\\
& (y, r, u, v) \in Y \leftarrow \text{generator operating constraints} && \text{(3d)}
\end{aligned}$$

Computational requirements of adaptive and non-adaptive models without any heuristics

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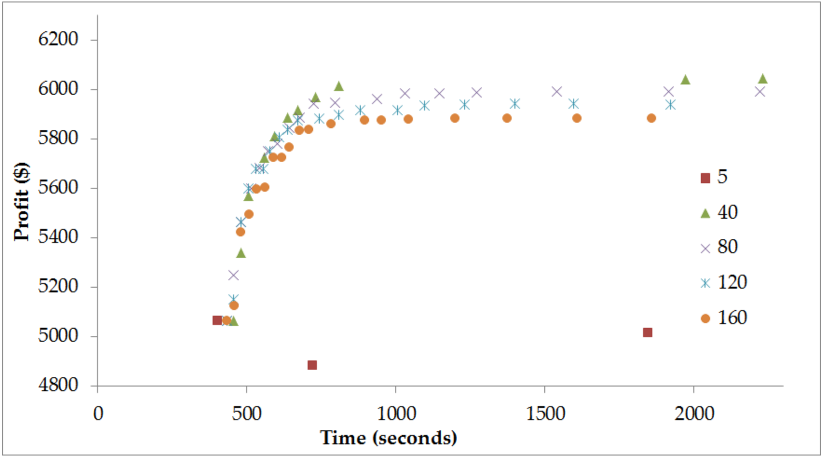
Scenarios	ε	Problem	Objective ($)	MIP Gap	Time (sec)
1500	0.05	Non-adaptive	2563.0	0%	3
1500	0.01	Non-adaptive	1889.5	0%	3
1500	0.05	Adaptive	3422.3	67.4%	2100
1500	0.01	Adaptive	3946.6	56.5%	2100

Algorithm

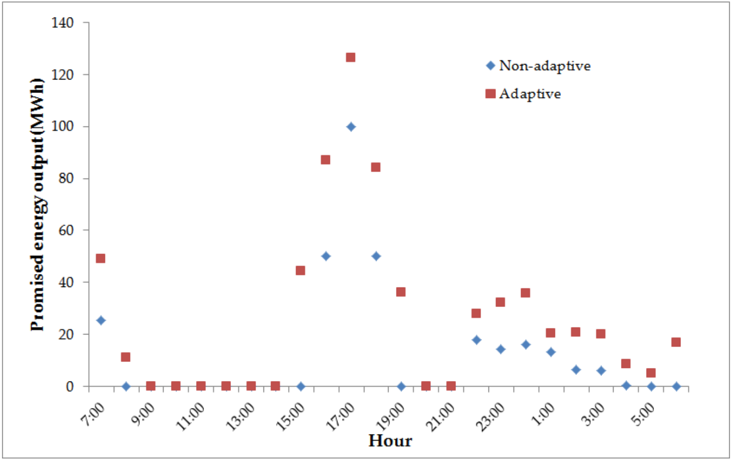
Algorithm Iterative Regularization with SAA

```
Input: $m, \delta, \rho$, $time$, 1500 i.i.d. realizations of $w$.
Output: $\hat{z}$: objective function value of original model with 1500 scenarios.
Generate $m$ i.i.d. realizations of $w$, and solve the SAA of original model to obtain $x_m^*$. Let $\hat{x} \leftarrow x_m^*$.
while time $\leq$ $time$ do
    Let $m \leftarrow \lceil m(1+\delta) \rceil$.
    Generate $m$ i.i.d. realizations of $w$, and solve the SAA of regularized model to obtain $x_m^*$. Let $\hat{x} \leftarrow x_m^*$.
    Solve original model with 1500 scenarios with $x$ fixed to $\hat{x}$, and let $\hat{z}$ denote the objective function value.
    Update time to the cumulative wall-clock time consumed so far.
end while
```

Result 1: $\rho = 40$ achieves the largest expected profit in the least time



Result 2: Adaptive model achieves synergy in solutions unlike the non-adaptive model



For the entire day, the non-adaptive model promises 300 MWh of energy while the adaptive model promises 625 MWh

Summary

- There is significant $ benefit to coupling a fast-moving energy source with a renewable source (adaptive model)

Summary

- There is significant $ benefit to coupling a fast-moving energy source with a renewable source (adaptive model)
- A slow-moving energy source and a renewable source could be looked as two separate assets (non-adaptive model)

We welcome collaborations with faculty, practitioners, and students!

Contents

Set of operating constraints for the generator, Y

$$y^{\omega}_{t+1} - y^{\omega}_{t} \leq \Delta(u^{\omega}_{t+1} + r^{\omega}_{t}), \forall t \in T \setminus \{|T|\} \tag{4a}$$

$$y^{\omega}_{t} - y^{\omega}_{t+1} \leq \Delta(v^{\omega}_{t+1} + r^{\omega}_{t+1}), \forall t \in T \setminus \{|T|\} \tag{4b}$$

$$\sum_{k=t-U+1}^{t} u^{\omega}_{k} \leq r^{\omega}_{t}, \forall t \in \{U, \dots, |T|\} \tag{4c}$$

$$\sum_{k=t-V+1}^{t} v^{\omega}_{k} \leq 1 - r^{\omega}_{t}, \forall t \in \{V, \dots, |T|\} \tag{4d}$$

$$u^{\omega}_{t} - v^{\omega}_{t} = r^{\omega}_{t} - r^{\omega}_{t-1}, \forall t \in T \tag{4e}$$

$$(G - g)r^{\omega}_{t} - (G - \Delta)u^{\omega}_{t} - (G - \Delta)v^{\omega}_{t+1} \geq y^{\omega}_{t}, \forall t \in T \tag{4f}$$

$$r^{\omega}_{t}, u^{\omega}_{t}, v^{\omega}_{t} \in \{0, 1\}, \forall t \in T \tag{4g}$$

$$y^{\omega}_{t} \geq 0, \forall t \in T \tag{4h}$$

Wind energy scenarios

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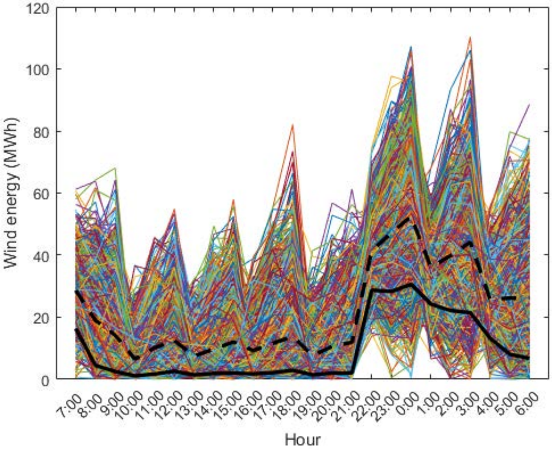

1500 hourly scenarios for wind energy generated using Monte Carlo sampling with a warm-up period of 140 hours. Dashed black line is median hourly value, and solid black line is 10th percentile.

Iterative regularization heuristic

Motivation:
Use regularization to help break symmetry and exploit knowledge of a potentially good solution

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Use regularization to help break symmetry and exploit knowledge of a potentially good solution

$$
\begin{aligned}
\max_{x,y,r,u,v} \quad & \sum_{t\in T} \left(R_t x_t - \mathbb{E}[B_t y_t^{\omega} + g r_t^{\omega}]\right) - \sum_{t\in T} \rho |x_t - \hat{x}_t| && \text{(5a)}\\
\text{s.t.} \quad & \mathbb{P}(y_t^{\omega} + g r_t^{\omega} + w_t^{\omega} \geq x_t, \forall t \in T) \geq 1 - \varepsilon && \text{(5b)}\\
& x_t \geq 0, \forall t \in T && \text{(5c)}\\
& (y^{\omega}, r^{\omega}, u^{\omega}, v^{\omega}) \in Y, \forall \omega \in \Omega && \text{(5d)}
\end{aligned}
$$

Iterative regularization heuristic

Motivation:
Use regularization to help break symmetry and exploit knowledge of a potentially good solution

$$\begin{aligned}
\max_{x,y,r,u,v} \quad & \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^{\omega} + g r_t^{\omega}]) - \sum_{t \in T} \rho |x_t - \hat{x}_t| && \text{(5a)} \\
\text{s.t.} \quad & \mathbb{P}(y_t^{\omega} + g r_t^{\omega} + w_t^{\omega} \geq x_t, \forall t \in T) \geq 1 - \varepsilon && \text{(5b)} \\
& x_t \geq 0, \forall t \in T && \text{(5c)} \\
& (y^{\omega}, r^{\omega}, u^{\omega}, v^{\omega}) \in Y, \forall \omega \in \Omega && \text{(5d)}
\end{aligned}$$

Unlike traditional regularization, we independently draw realizations for SAA at each iteration

Adaptive model: Big *M* formulation for SAA

$$x_t \leq y_t^{\omega} + gr_t^{\omega} + w_t^{\omega} + M_t^{\omega} z^{\omega}, \forall t \in T, \omega \in \Omega \tag{6a}$$

$$\sum_{\omega \in \Omega} z^{\omega} \leq \lfloor N\varepsilon \rfloor \tag{6b}$$

$$z^{\omega} \in \{0, 1\}, \quad \forall \omega \in \Omega \tag{6c}$$

Non-adaptive model: Extended variable formulation for SAA

First, sort the wind scenarios for each t: $w_t^{\omega(1,t)} \leq \cdots \leq w_t^{\omega(N,t)}$

$$x_t \leq y_t + w_t^{\omega(1,t)} + \sum_{\ell=1}^{N-1} \left(w_t^{\omega(\ell+1,t)} - w_t^{\omega(\ell,t)} \right) q_t^{\omega(\ell,t)}, \quad \forall t \in T \tag{7a}$$

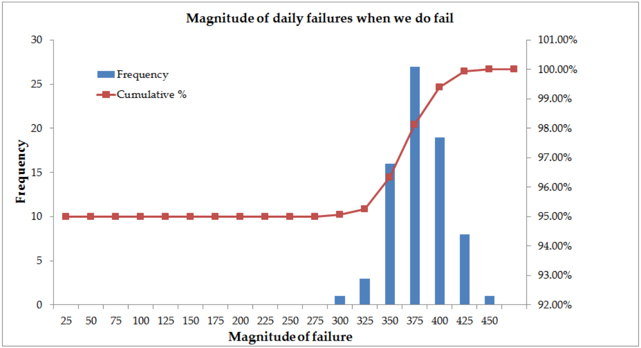
$$q_t^{\omega} \leq z^{\omega}, \ \forall \omega \in \Omega, t \in T \tag{7b}$$

$$q_t^{\omega(\ell+1,t)} \leq q_t^{\omega(\ell,t)}, \ \forall \ell = 1, 2, \ldots, N-2, \forall t \in T \tag{7c}$$

$$\sum_{\omega \in \Omega} z^{\omega} \leq \lfloor N\varepsilon \rfloor \tag{7d}$$

$$q_t^{\omega}, z^{\omega} \in \{0, 1\}, \ \forall \omega \in \Omega \tag{7e}$$

Magnitude of failures



For the 75 failed scenarios ($\varepsilon = 0.05$), magnitude of average daily-failure is 368MWh

Result 3: $\rho = 40$ achieves a statistically larger expected profit than other ρ values

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- Using 10 i.i.d. batches of 1500 scenarios, reject the null hypothesis (that expected profit under $\rho = 40$ is at most that under $\rho = 80$) with a p-value of $p = 0.999$
- Using same 10 batches, 95% confidence interval on expected profit with $\rho = 40$ is [6069.8, 6123.2]