Evaluation of the Impact of Intra-Day Distributed PV and Wind Generation Forecasts on Decision-making in the Operations of an Island Grid System

John W Zack
UL AWS Truepower
Albany, NY 12205
jzack@awstruepower.com

Robert Kaneshiro and Lisa Dangelmaier
Hawaii Electric Light Company
Hilo, Hawaii
robert.kaneshiro@hawaiielectriclight.com

***Abstract*—This paper provides an overview of an in-progress study to identify which aspects of intra-day forecast performance are most critical to providing value for the management of the impact of renewable generation variability on an island system with no interconnections and a high penetration of variable renewable generation. The objective is to identify which forecast information provides the value to operational decision-making and to design a customized forecast evaluation metric that more effectively measures the sensitivity of the operational decision-making environment to forecast error than traditional error metrics.**

The platform for the study is the island grid system operated by the Hawaii Electric Light Company (HELCO) on the "Big Island" in the State of Hawaii. Forecasts from a customized wind and solar forecast system have been used in the operational decision making process for several years but the quantification of the actual value of forecast information has been difficult. A customized forecast evaluation system is being built from (1) the identification of the critical time periods and scenarios as well as the key parameters that impact operational decisions at those times, and (2) the formulation a forecast evaluation metric that emphasizes the performance for the prediction of key parameters during the critical time periods and scenarios.

The initial phase of this project has identified three key daily time periods with characteristic operating issues. A categorical forecast structure has been developed to focus on the key information for each of the three key decision-making periods. A generalized skill score has been defined to evaluate the categorical forecasts in a way that emphasizes performance in infrequent but key scenarios.

Keywords-energy forecast value; grid management with high renewable pentration;wind and solar integration; renewable

I. INTRODUCTION

The increasing penetration level of non-dispatchable variable renewable generation resources such as wind-based and solar-based generators on grid systems have created the need for tools and approaches to assist grid operators in the management of the variability in order to maintain the supply-demand balance and reliability in an economical manner. The need increases as higher amounts of variable resources increases offline cycling of conventional units, unit commitment decisions, and uncertainty in the management of energy resources. In addition to online reserves (which have cost implications), there are a number of flexible energy resources helpful to grid operators managing variability including: (1) energy storage, (2) demand response, (3) active power control of variable generation resources, and (4) flexible, quick-start generation resources. Short-term forecasting of renewable generation variability provides useful insights into the best use of these energy resources to meet reliability and cost goals through more optimized use of the available energy resources. Some of these resources are not available on specific grid systems, due to the limitations of the current mix of system resources and the high cost or lengthy time to make changes to system assets. Short-term forecasting typically has a low implementation barrier and a very favorable cost/benefit ratio. However, the simple availability of forecast data to the grid operator does not guarantee that the potential value of that information will be realized in the grid management process. A key to realizing value from forecast information is the extraction of components that address specific operational issues and inform associated decision-making. It is critical to evaluate and quantify how well the forecast information addresses key operational issues, to instill confidence in the forecast users.

Island grid systems with high variable penetration often have the most acute need for tools to manage the impact of variable generation because of their (1) small system size and therefore high sensitivity to variations in load and generation, and (2) lack of interconnections to buffer supply-demand imbalances. Thus, they are an excellent venue to develop and evaluate methods to optimize the value of forecast information in operational decision-making.

This study addresses the issue of which aspects of intra-day forecast performance are most critical to providing value to the management of the impact of renewable generation variability on an island system with a high penetration of variable renewable generation. The objective is to identify which forecast information provides the value to operational decision-making and to design a customized forecast evaluation metric that more effectively measures the sensitivity of the operational decision-making to forecast error than traditional error metrics.

The venue for this investigation is the island of Hawaii, which is also known as the Big Island because it is the largest island in the island chain that comprises the State of Hawaii. The electric system on the island is operated by the Hawaii Electric Light Company (HELCO). HELCO is a subsidiary of Hawaiian Electric Company (HECO), which operates the grids on five of the eight islands of the state of Hawaii.

II. THE OPERATING ENVIRONMENT

The typical profile of the generation assets on the HELCO system is presented in Table I. While units operated 24 hours are labeled as "base", in actuality all generation is required to load follow and deep cycle except the 38 MW of geothermal generation, which has a normal minimum of 27-30 MW. Utility-scale variable renewable generation resources on the system are listed in Table II. These provide 38.2 MW of "must take as available" generation that consists of 31 MW of wind generation and 16.2 MW of hydro generation. The output from these must-take resources can be reduced during low demand, but only after non-essential generators are cycled offline and reserves down are at minimum.

In addition to the system-level resources in Table I, there is also approximately 90 MW of "behind-the-meter" distributed (mostly residential and commercial rooftop) PV generation that is mostly not visible (measurable) nor controllable by HELCO. As a result of the combined impact of the utility-scale must-take resources and distributed PV, the system operator must make significantly more unit commitment decisions. Generators that were previously scheduled have been retired, and generators that were operated continuously are subject to offline cycling once or twice daily. This change in resources has created much more variability in the demand to be served while at the same time increased the need for demand forecasting to make unit commitment and decommitment decisions.

TABLE I. GENERATION RESOURCE PROFILE OF THE HELCO SYSTEM. INDEPENDENT POWER PRODUCERS (IPPS) ARE DENOTED BY BOLD, ITALICS TEXT

Base (24-hr) Units
Hill 5 & 6 Steam Units
Keahole 1CT in combine cycle (CC)
PGV (Geothermal)
Intermediate Units
Keahole 2nd unit in CC
HEP 1st and 2nd in CC
Peaking/Emergency Units
Kanoelehua CT-1
Keahole CT-2
Puna CT-3
Puna Steam Unit
12-Small Diesel Generators
As-Available Must-Take
HRD Wind farm (10.5 MW)
Pakini Nui Windfarm (20.5 MW)
Wailuku River Hydro (12 MW)
Puueo Hydro (3.1 MW)
Waiau Hydro (1.1 MW)

TABLE II. SUMMARY OF THE RENEWABLE GENERATION RESOURCES ON THE HELCO SYSTEM GENERATION

Type of Resource	Capacity
Geothermal	38 MW
Hydro (3 facilities)	16.2 MW
Wind (2 facilities)	31 MW
Solar (distributed behind the meter)	90 MW

The average net and gross load profiles as well as the difference (Gross-Net) for the weekdays during each quarter of 2017 is depicted in Figure 1. The "net load" is the measurable demand served by the HELCO generation resources and incorporates the behind the meter PV generation that offsets some of the actual load during the daylight hours. The "gross load" is the true demand by users on the system without the offsetting behind the meter PV generation. The gross load is inferred by adding the estimated PV generation to the measured net load. Therefore, the difference between the gross and net load (shown in the bottom panel of Figure 1) is the estimated system-wide PV production. The average net load profiles indicate that there are four significant daily features that define the daily system management cycle: (1) a nighttime minimum between midnight and 0900 HST, (2) a daytime (morning) peak between 0600 HST and 1300 HST, (3) a daytime minimum between 0900 HST and 1400 HST due to distributed PV generation, and (4) an evening peak between 1300 HST and midnight. During 2017 the lowest net load of 84.8 MW occurred during the daytime minimum period on March 19 due to the very high behind the meter PV production on that day. This was slightly lower than the lowest nighttime minimum of 86.1 MW that occurred on December 15. The highest net load of 190.5 MW occurred during the evening peak on October 23.

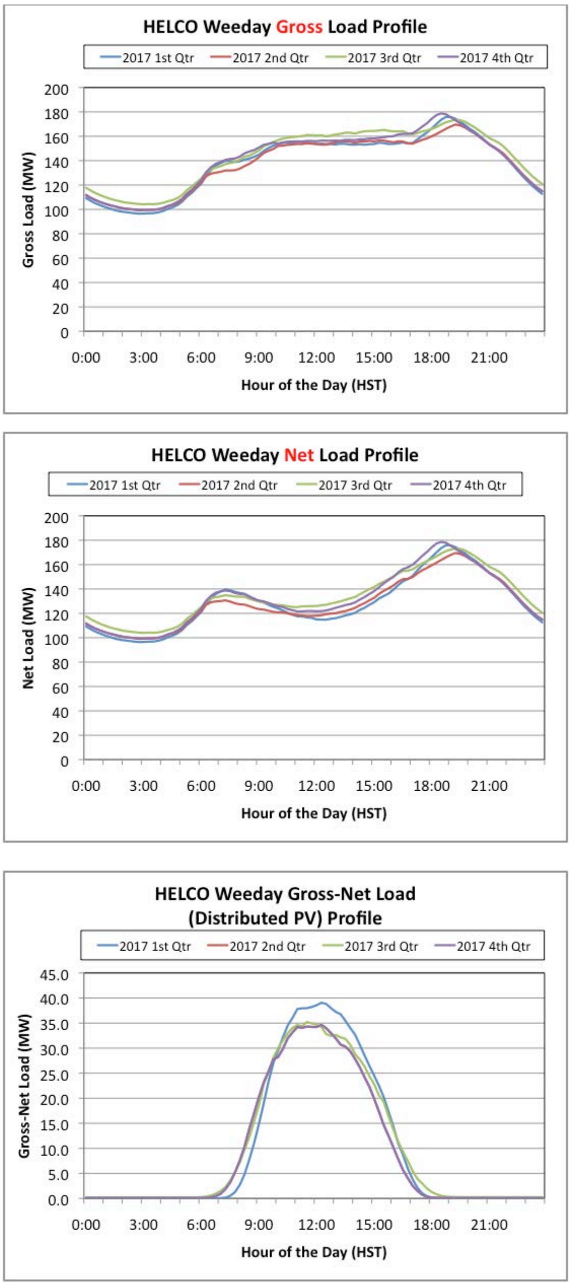

Figure 1. 2017 quarterly average gross (top) and net (middle) load profiles and their difference (bottom) for the HELCO system.

There are no interconnections between the electric grids on any of the islands that comprise the state of Hawaii. Therefore, the HELCO system operates without the ability to export or import power from neighboring systems, which of course increases the difficulty in managing generation or demand variability. All balancing must be done by the resources available on the HELCO system. Any imbalance results in a system frequency excursion.

III. KEY OPERATING ISSUES

The shapes of the gross load and solar generation profiles, along with the potential for significant short-term variability of the solar-based and wind-based generation and the attributes of the non-renewable generation resources, combine to create a set of ongoing operating issues that are characteristic of specific times of the day. This section presents three key issues and the role that solar and wind forecasting can play in the management of those issues.

A. *Morning Distributed PV Rise*

The first critical time of day is typically before sunrise at about 0500 HST. The challenge here is to determine if midday net loads will be low enough to shutdown a unit after the morning peak. If so, a simple cycle CT that has no start/stop restrictions but less efficient can be used to serve the morning peak then shutdown when no longer needed. If net loads are expected to remain high, a more efficient combine cycle CT will be used throughout the day. One of the combine cycle plants has a permit and contract start/stop restriction, and is not allowed to have multiple starts in a calendar day.

A second issue at this time is whether an excess energy situation is expected due to high “as available” (those listed in Table I) generation. The forecasting of the duration of the expected excess energy event is needed to determine whether curtailment of the as-available renewable generation or taking a unit offline will best address the situation. Unit will be taken offline if the excess energy event duration is greater than the minimum downtime of the unit.

Wind and solar generation forecasts available at 0500 HST for the middle of the day are important factors in the pre-sunrise decision-making. There are two key forecasting questions for the midday period: (1) will the distributed solar generation rise to its typical midday values or will the weather conditions be much cloudier than usual and this result in much below normal solar generation? (2) will the wind generation increase or decrease from its pre-sunrise level and thus either contribute to or offset a potential excess energy situation?

B. *Preparation for the Evening Demand Peak*

Another important decision-making time is typically in the early afternoon (~1300 HST) when plans have to be made to position the system for the evening peak demand. The key forecasting issues at this time are (1) will the distributed solar production decrease at a typical rate as evening approaches or will it decrease more quickly than the average rate (i.e. more late afternoon clouds than typical)? and (2) will wind remain constant, increase or decrease as the evening demand peak approaches?

The starting of combine cycle combustion turbines requires the unit be kept at constant load while the heat recovery steam generator is started, limiting the system's regulating capability. Each unit added increases the system minimum dispatch limit. An ability to anticipate the trend and variability of the wind can help to decide the timing of when to start bringing the units online.

These issues are illustrated in Figure 2 by a typical time series of wind and solar generation, net load and the upper and lower range of the regulating reserve on the HELCO system from early afternoon through the evening demand peak. This chart indicates that while the Keahole 2nd unit is starting-up, the regulating reserve range becomes tighter. This range must include contingency reserves, which makes the range tighter. Note that the start initiation for a combine cycle unit occurs 25-40 minutes before the unit first comes online. More accurate forecasts of wind and solar generation during this time period enable a better timing of the unit start-ups.

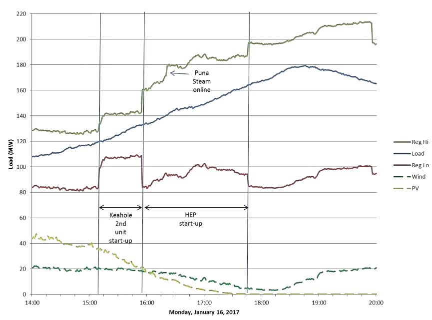

Figure 2. An example of a typical time series of wind and solar generation, net load and the upper and lower range of the regulating reserve on the HELCO system from early afternoon (1400 HST) through the evening demand peak (2000 HST).

C. *Midday Net Load Ramps*

A third operating issue that must be considered is the probability of the occurrence of sudden large amplitude changes (i.e. ramps) in distributed solar production during the midday period (0800 HST to 1400 HST) that induce large ramps in the net load. An example of such a day (December 31, 2016) is shown in Figure 3. On this day, intra-hour net load ramps of approximately 20 MW (15% to 20% of the net load) were observed during the midday period. It should be noted that the ramping behavior began early in the day (about 0900 HST) but ceased at approximately 1400 HST. This makes the point that the characteristics of the net load/PV ramping behavior can vary considerably during the peak PV production period (0900-1500 HST).

In these situations it is important to have adequate ramping capability available with the online units to ensure that the system frequency doesn't go too high or too low. As the PV penetration on the system increases, it is expected that the frequency and amplitude of these type of events will increase. Therefore, the ability to anticipate the periods for which the probability of this type of behavior is high will have considerable value to the grid operators to manage reserve capacity and ramping rate requirements.

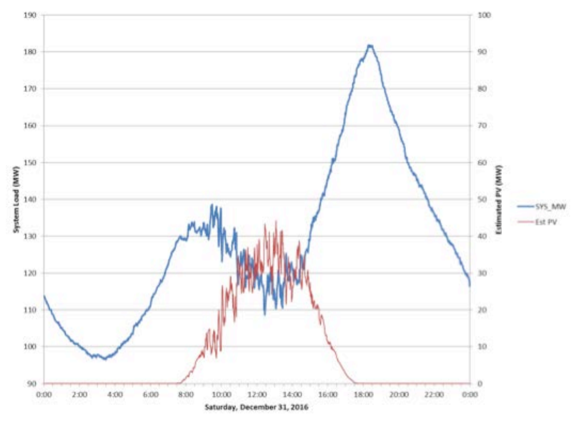

Figure 3. The time series of measured net system load and estimated distributed PV generation for a day (December 31, 2016) characterized by large midday net load ramps induced by PV generation ramps.

IV. RENEWABLE GENERATION FORECAST SYSTEM

Renewable generation forecasts are provided to HELCO by a customized prediction system called the Solar and Wind Integration Forecast Tool (SWIFT) [1]. SWIFT is based on the multi-method ensemble approach to forecasting. In this approach, forecasts are generated by multiple forecast algorithms. The ultimate forecast that is delivered to the user is then created by statistically constructing a deterministic or probabilistic composite of the individual forecasts.

SWIFT provides wind and solar forecasts on two different look-ahead time scales. The first is a 6-hour look-ahead period with a forecast increment of 15 minutes that is updated every 15 minutes. This product is targeted for the type of intra-day decision-making described in the previous section. The second look-ahead time frame is 168 hours (7 days), with a 1-hour forecast increment that is updated on an hourly basis. This is targeted for longer term planning activities. The forecast content is the same for both look-ahead time periods.

Solar generation forecasts are provided for each utility-scale facility (there are none at present on the Big Island but some are planned) and for the aggregate of all distributed PV generation resources connected to each substation. Regional and system-wide forecasts are then produced by combining the forecasted production from the substations and utility-scale facilities. Wind generation forecasts are provided for each utility-scale facility. These are combined to produce a system-wide wind generation forecast.

An example of a SWIFT 6-hour ahead system-wide solar forecast is shown in Figure 4. This forecast was issued at 0500 HST on October 24, 2017. The forecast is expressed in terms of nine probability of exceedance (POE) values. This implicitly provide a modest representation of the probability density function at each look-ahead time. The 50% POE value is often used as a deterministic forecast.

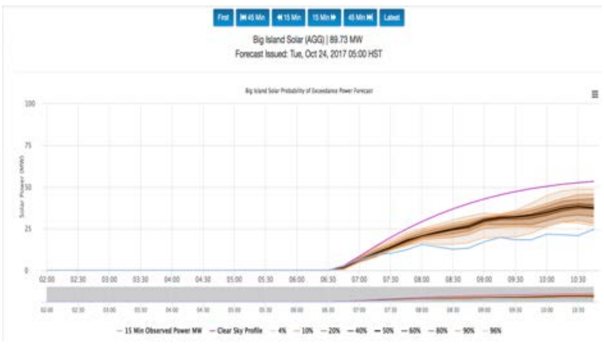

Figure 4. The distributed solar generation (MW) forecast for the HELCO system produced by SWIFT at 0500 HST October 24, 2018. The purple line represents the clear sky generation profile. The blue line is the estimated actual generation (calculated historically at the end of forecast period). The brown lines are the probability of exeedance (POE) forecasts from SWIFT.

V. Case Examples of Significant Morning Planning Issues

Three examples of the impact of generation and load variability during the morning and midday period on HELCO grid operations are provided in this section. These cases are examples of the issues associated with the morning planning for the morning demand peak and the midday minimum period as presented in Section 3.1. Accurate forecasts of key attributes of the wind, solar and load variability had the potential to provide value in all cases. However, the accuracy of the key aspects of the forecasts was not adequate in all of the cases to realize the potential value.

A. *Oct 24, 2017: High Daytime Net Load Peak*

The net load, distributed solar production and wind generation for this day are shown in Figure 5. This day was characterized by high net loads throughout the daytime hours. The morning daytime net load peak was an unusually high 172 MW. The net load remained unusually high throughout the middle of the day with the midday minimum near 150 MW (versus an average value of about 120 MW shown in Figure 1). The high net load values were caused by two weather-related factors: (1) the actual load was above normal because of the warm conditions with much higher than normal humidity over the entire island which increases the cooling-related demand, and (2) very low distributed PV production due to atypical widespread and thick cloud cover. Furthermore, wind generation was near zero most of the day and therefore did not help to offset the high loads.

The high net loads were not anticipated. This was partially due to over-forecast of the midday solar generation in the morning forecast. This can be seen in the 0500 HST system-wide solar forecast for this day that is shown in Figure 4. The forecast system actually indicated that it would be a relatively cloudy day with much below average solar production but the clouds were thicker and more widespread than anticipated and therefore the production was substantially below the forecast envelope. A second factor was the higher than average gross load due to the high humidity. Gross load forecasts are not part of the SWIFT product set but accurate weather-dependent gross load forecasts would also have contributed to more economical decision-making on this day. In

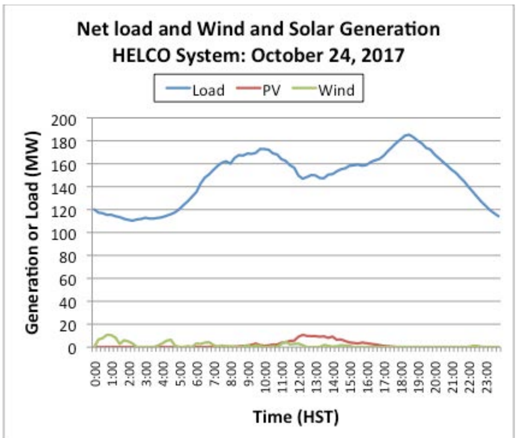

Figure 5. Time series of net load, distributed PV production and wind generation on the HELCO system for October 24, 2017.

addition to the error in the distributed PV forecast and the lack of a weather dependent gross load forecast, the morning wind forecast indicated that the wind production would be higher than it actually was. This made it more difficult for the operators to anticipate the needed generation. Therefore, the system has to rely on less economical fast starting simple cycle combustion turbines and quick start diesels. Had high loads been anticipated, a combine cycle combustion turbine would have been utilized.

B. *March 19, 2017: Low Daytime Net Loads*

The net load, distributed solar production and wind generation for this day are shown in Figure 6. In contrast to the conditions experienced on October 24, the conditions on March 19, 2017 were characterized by extremely low daytime loads. In fact, the daytime minimum load was lower than the nighttime minimum load. This was due to the combination of extremely high system-wide distributed PV production due to clear conditions over the island and a below average daytime gross load due to cooler and drier than normal conditions.

When loads are low (below about 120 MW), curtailing of variable renewable generation may be necessary if intermediate generating units are not taken offline. However, intermediate generating units are only taken offline if they are forecasted to not be needed for at least 3 hours (typical minimum downtime). On March 19, the morning forecasts indicated a high PV day (probably with minimal ramps), and wind generation increasing. This gave confidence to the operator to take a combine cycle unit offline to avoid curtailment of the as-available renewable generation.

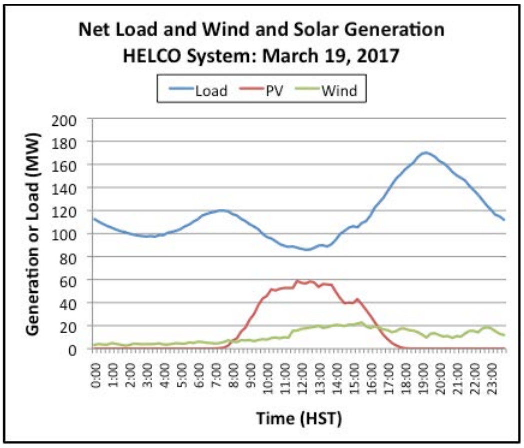

Figure 6. Time series of net load, distributed PV production and wind generation on the HELCO system for March 19, 2017.

Thus on this day an accurate morning forecast of high PV production and increasing wind production during the day allowed all intermediate units to be taken offline to avoid the curtailment of the as-available renewable generators.

C. *Feb 7, 2017: Decreasing Wind Production*

The February 7, 2017 case was an example of a day with a fairly typical gross load and PV production profile but with unexpected behavior of the wind generation during the morning and midday hours. The net load, distributed solar production and wind generation for February 7 are shown in Figure 7. The morning forecast for the distributed solar production profile was quite good. However, the morning wind forecast indicated that the total wind generation (from the two facilities) would remain above 16 MW as it had been during the pre-sunrise hours. However, the actual winds decreased while the morning load was increasing. Fortunately, the operator started a combine cycle unit early enough and had fast start units available to handle the change in generating capability.

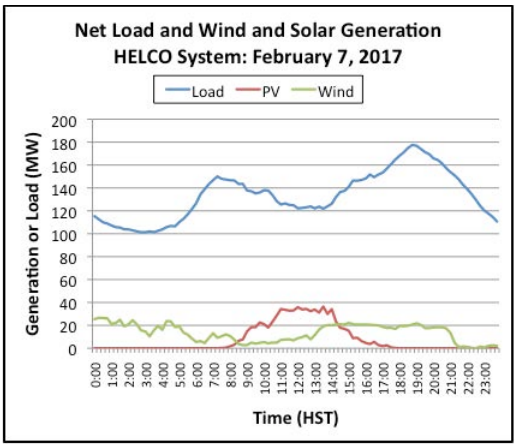

Figure 7. Time series of net load, distributed PV production and wind generation on the HELCO system for February 7, 2017.

VI. EVALUATION OF FORECAST PERFORMANCE

As noted previously, the ultimate objective of this research effort is to identify the key attributes of the intra-day wind and solar forecast that facilitate better decisions by the operators at critical decision-making times during the daily cycle, and to develop metrics that can effectively evaluate the performance of the forecasts for those attributes and time periods. In the preceding section, three key time frames and wind and solar forecast issues were identified for the HELCO system. However, the focus during the first phase of the project is on the early morning planning to serve the morning demand peak and the midday net load minimum.

The key point is that the small-scale details of the forecast often do not matter in the decision-making process. The issue is whether the key attributes that impact the decision that has to be made at a given time are basically correct (i.e. the big picture). For the morning planning process the key issues are (1) whether the distributed solar production will rise to a typical midday value or will it be well below or above average, and (2) will the wind generation continue at its current level or significantly increase or decrease during the morning demand peak and into midday.

Based on this perspective, an event-based categorical forecast scoring system was defined. For the morning forecast three categories of wind and three categories of solar events were defined. These are listed in Table III. The most operational value is obtained when both the wind and solar trends are correctly anticipated. A total of 9 forecast categories are obtained by pairing each solar category with each of the wind categories (for example, solar category 1S with wind categories 1W, 2W and 3W yields three composite categories). This can then be transformed into a 9 by 9 forecast vs. outcome contingency matrix, which maps the relationship between the forecasted categories and the associated observed outcomes. The contingency matrix forms the basis for an evaluation metric.

TABLE III. SOLAR AND WIND EVENT FORECAST CATEGORIES FOR MORNING OPERATIONAL PLANNING BASED ON 0500 HST FORECAST

Solar Event Categories	Wind Event Categories
1S: 08-11 HST Solar Ramp Rtae Significantly Below Average	1W: Significant 06-11 HST Wind Gen Decrease
2S: 08-11 HST Solar Ramp Rate About Average	2W: No Significant 06-11 HST Wind Gen Change
3S: 08-11 HST Solar Ramp Rate Significantly Below Avg	3W: Significant 06-11 HST Wind Gen Increase

A simple accuracy metric can be formulated by defining $n(F_i,O_j)$ as the number of forecasts in category i that have an outcome of category j. If N is the total number of forecasts and K is the number of forecast categories then a “hit rate” (HR with “hit” meaning a successful forecast) can be defined as:

$$HR = \frac{1}{N}\sum_{i=1}^{K} n(F_i,O_i) \tag{1}$$

This is simply the ratio of all of the correct forecasts (i.e. the forecast and outcome category are the same) to the total number of forecasts. However, the problem with this metric is that it will be weighted by the frequency of occurrence of each category. So the score will be determined mostly by the most common categories (such as no change in wind and a typical solar morning). This is not satisfactory since it is most important to have accurate forecasts for the unusual situations. Thus, it is desirable to place greater weight on those situations in the evaluation metric. This can be done through the use of a generalized skill score (GS) in a manner similar to that formulated by [2] and [3]:

$$GS = \frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{K} n(F_i,O_j)s_{ij} \tag{2}$$

This metric differs from the simple accuracy metric by considering all forecast-outcome combinations (i.e. all cells in the 9 by 9 contingency matrix) and also by the use of a scoring parameter (S_{ij}) that weights the contribution of each matrix cell in the calculation of the overall metric. The scoring metric is formulated to produce a higher score for successful forecasts of less frequent events and to provide some credit for incorrect forecasts based on how far they are from the outcome category. This credit can be adjusted to account for the sensitivity of the decision-making process to a particular type of error. Thus, incorrect forecasts for critical scenarios can be penalized more than incorrect forecasts in less critical scenarios.

An example of the data for the solar component of the categorical forecast scheme for October, 2017 is shown in Figure 8. This shows the forecasted (horizontal axis) and actual/outcome (vertical axis) data for the 0800 to 1100 HST change in system-wide distributed PV production. The diagonal red line denotes the set of points for which the forecasted and actual values are the same (i.e. perfect forecasts). The further a point is from that line, the larger the forecast error. It can be easily seen that the forecasts are best for the typical outcome values (i.e. when the outcome values are in the middle of their range, the points cluster around the red line). However, when the outcomes are near the ends of the (vertical) range the points are typically further from the red line, which indicates larger forecast errors. Widely used forecast performance metrics such as the mean absolute error (MAE) or root mean square error (RMSE) are dominated by the large cluster of points in the middle of the outcome distribution. So the forecast performance appears to be quite good for this sample. However, the forecasts of the infrequent but operationally critical events are not nearly as good. The most vivid examples are the October 23 and 24 cases, which were previously noted as being operationally significant days. These were cloudy days with very low PV production. The targeted evaluation approach will focus on the performance during atypical situations that are operationally significant.

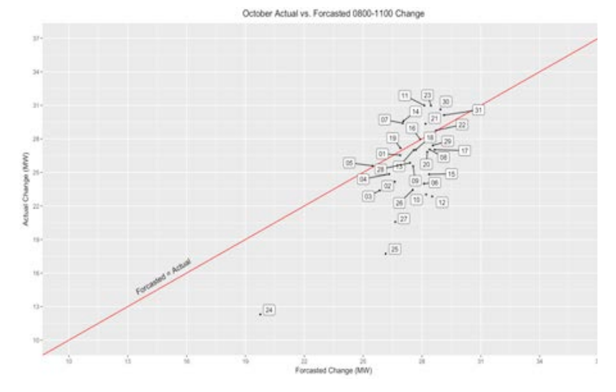

Figure 8. Scatter plot of each day's forecasted vs. observed increase in system-wide PV generation between 0800 and 1100 HST for October 2017. The day of the month is denoted for each point.

VII. SUMMARY

A study is in progress to address the issue of which aspects of intra-day forecast performance are most critical to providing value for the management of the impact of renewable generation variability on an island system with no interconnections and a high penetration of variable renewable generation. Its objective is to identify which forecast information provides the value to operational decision-making and to design a customized forecast evaluation metric that more effectively measures the sensitivity of the operational decision-making environment to forecast error than traditional error metrics.

The platform for the study is the island grid system operated by the Hawaii Electric Light Company on the "Big Island" in the State of Hawaii. The system has a high penetration of renewable generation with a total renewable (geothermal, hydro, wind and solar) capacity of approximately 175.2 MW on a system whose net load ranged from 85 MW to 190 MW during 2017. Highly variable renewable resources (wind and solar) comprise about 121 MW of the 175.2 MW capacity. This high penetration of variable renewable resources on a system with no interconnections poses a number of operating issues that have to be addressed to maintain the balance of supply and demand as well as grid stability.

In addition to other tools, intra-day forecasting is being used to assist in the management of the impact of wind and solar variability on the grid system. Traditional forecast error metrics such as MAE or RMSE have been used but these weight all forecasts equally, and most of the forecast value is concentrated in forecasts made at key times of the day in critical scenarios. A customized forecast evaluation system is being built from (1) the identification of the critical time periods and scenarios as well as the key parameters that impact operational decisions at those times, and (2) the formulation a forecast evaluation metric that emphasizes the performance for the prediction of key parameters during the critical time periods and scenarios.

The initial phase of this project has identified three key daily time periods with characteristic operating issues. A categorical forecast structure has been developed to focus on the key information for each of the three key decision-making periods. A generalized skill score has been defined to evaluate the categorical forecasts in a way that emphasizes performance in infrequent but key scenarios.

ACKNOWLEDGMENT

The authors thank the HELCO operational team for their extensive feedback on the benefits and issues in their use of wind and solar in grid management decision-making.

REFERENCES

[1] J. W. Zack, J. D. Freedman, and D. Nakafuji, "A solar and wind integrated forecast tool (SWIFT) designed for the management of renewable energy variability on island grid systems", Proceedings of the 3rd International Solar Integration Workshop, London, 2013, Paper SIW13-1075.

[2] J.D. Gordon, "Evaluating the skill of categorical forecasts". *Mon Wea Rev*, vol.110, 1982, pp. 657-661

[3] J.P. Gerrity, "A note on Gandin's and Murphy's equitable skill score." *Mon Wea Rev*., vol. 120, 1992, pp. 2709-2712.